Non-factorizable Contribution to B_K of Order $\langle G^3 \rangle$

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Abstract

Within the Chiral Quark Model we have calculated the effect of non-factorizable $\langle G^3 \rangle$ contribution to the B_K parameter. When all diagrams are summed, a vanishing result is obtained.

1 Introduction

The low energy effects of $K^0 - \overline{K}^0$ mixing has been a subject of a lot of discussion in the literature. See for example [1],[2], [3] and references therein. Its amplitude is parametrized by the B_K parameter and is conventionally defined by

$$\langle K^0 | \mathcal{O}(\Delta S = 2) | \overline{K}^0 \rangle = \frac{4}{3} f_K^2 m_K^2 B_K \tag{1}$$

The vacuum saturation approach accomodates only a factorizable contribution which yields $B_K = 3/4$ in the large N_c limit. By adding the factorizable next to leading order contribution $1/N_c$ increases B_K to 1. Lattice calculations seem to indicate that the physical B_K is numerically around 0.7. Non-perturbative and non-factorizable effects can be taken into account by dressing the Feynman diagrams with soft external gluons in all possible ways. In doing this one makes for instance the identification $G^a_{\mu\nu} G^{a,\mu\nu} \to \langle G^2 \rangle$ where $G_{\mu\nu}$ is the gluon tensor of dimension two. The coefficient of the $\langle G^2 \rangle$ term has been calculated by Pich and de Rafael [4]. They got a significant contribution to B_K. Numerically, their result change very much the next to leading $1/N_c$ contribution. However, due to large uncertainties in the experimental value of the condensate $\langle G^2 \rangle$, B_K is determined to lie between 0.3 and 0.6. The purpose of this paper is not to find a better numerical determination of B_K, but rather to find a more general expression of B_K in terms of non-perturbative $\langle G^3 \rangle$ effects.

In this paper we use the Chiral Quark Model with a constituent quark mass M to calculate the amplitude. This model introduces interaction terms between quarks and mesons. This means that hadronic matrix elements can be calculated as loop diagrams.

When calculating fermion propagators in an external field we use the point gauge prescription in order to get results which directly contain gauge invariant quantities.

There are two independent condensates with dimension six, namely $\langle G^3 \rangle$ and $\langle j^2 \rangle$. Since we are working in the heavy quark limit, $\langle j^2 \rangle$ will be given as an expansion in gluon condensates of higher order and inverse proportional to the heavy quark mass M [5] i.e.

$$\langle j^2 \rangle \propto \frac{\langle G^4 \rangle}{M^2} + \cdots . \tag{2}$$

To consider $\langle j^2 \rangle$ terms we have to consider direct $\langle G^4 \rangle$ terms as well. Therefore, our object of investigation is the $\langle G^3 \rangle$ terms only.

2 The Chiral Quark Model

In the Chiral Quark Model, chiral symmetry breaking is taken into account by adding an extra term $\mathcal{L}_\chi$ to the standard QCD Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_\chi \tag{3}$$

where

$$\mathcal{L}_{QCD} = \overline{q}(i\gamma \cdot D - \mathcal{M}_q)q - \frac{1}{4}G^{a,\mu\nu}G^a_{\mu\nu}. \tag{4}$$

and the $\mathcal{L}_\chi$ term is only taken into account for energy/momenta of order Λ_χ and below. The q-fields denotes only u, d, s quarks. The chiral symmetry breaking scale is $\Lambda_\chi = 0.83\ GeV$ and

$$\mathcal{L}_\chi = -M\,(\overline{q}_L\Sigma q_R + \overline{q}_R\Sigma^\dagger q_L). \tag{5}$$

The prefactor M is interpreted as the constituent quark mass and is of the order $200 - 300\ MeV$. The pseudoscalar Goldstone-octet fields π^a are contained in

$$\Sigma = exp(i\sum_a \lambda^a\pi^a/f) \tag{6}$$

In the limit where $\mathcal{M}_q \to 0$, the model is $SU(3)_L \otimes SU(3)_R$ invariant.

The chiral symmetry is thus realized through the existence of pseudoscalar Goldstone bosons.

This model connects mesons and quarks and one can calculate hadronic matrix elements as loop diagrams. By expanding Σ one finds the coupling between a quark density and a meson. For the $\overline{q}qmeson$ vertex, the relevant coupling constant is $\propto M\gamma_5/f$ where f is associated with the physical pion decay constant and the proportionality factor is given by $SU(3)$ symmetry. In the following, mesons are drawn by dotted lines while quarks are drawn with solid lines.

In the $SU(3)$ limit without gluonic condensates the pion decay constant is defined by

$$f_\pi \;=\; \frac{N_c M^2}{4\pi^2 f}\hat{f}_\pi \tag{7}$$

$\propto$ "  " $(vertex : \gamma^\mu L)$ (8)

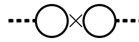
With a cut-off Λ, the explicit expression for $\hat{f}_\pi = ln(\Lambda^2/M^2) + \cdots$ while in dimensional regularization $\hat{f}_\pi = (2 - D/2)^{-1} + \cdots$. Λ is of the order of the chiral symmetry breaking scale Λ_χ. Although f_π and f appear differently in the Chiral Quark Model, they are for practical purposes numerically equal ($f_\pi = 93.3\ MeV$).

3 Diagrammatic Calculation of $K^0 - \overline{K}^0$

The lowest order local operator, of dimension six, contributing to $K^0 - \overline{K}^0$ is

$$\mathcal{O}(\Delta S = 2) = (\overline{d}\gamma_\mu Ls)(\overline{d}\gamma^\mu Ls). \tag{9}$$

This operator gives rise to two diagrams pictorially given as

$\langle K^0|\mathcal{O}(\Delta S = 2)|\overline{K}^0\rangle =$ "  +  " (10)

The first diagram is $\propto N_c^2$ while the second diagram is colour suppressed and is $\propto N_c$.

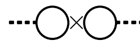
There are also other contributions than eq.10 with derivatives acting on the quantum fields. They are generated from the siamese penguin operator [6], [7] or similar non-local operators.

In some calculations it is customary to Fierz transform the operator $\mathcal{O}$. We can take some advantages of the transformed operator as some of the generated diagrams cancel. Consider the Fierz transformed operator:

$$\mathcal{O}_F = \frac{1}{N_c}(\overline{d}\gamma_\mu Ls)(\overline{d}\gamma^\mu Ls) + 2(\overline{d}\gamma_\mu Lt^a s)(\overline{d}\gamma^\mu Lt^a s) \tag{11}$$

The matrix element can now be written symbolically:

$\langle K^0|\mathcal{O}_F|\overline{K}^0\rangle \;=\; \frac{1}{N_c}\Big\{$"  $+$  "$\Big\}\,(vertex:\gamma^\mu L)$

$+2\Big\{$"  $+$  "$\Big\}\,(vertex:\gamma^\mu Lt^a)$ (12)

The Fierz transformed operator gives rise to four diagrams. The symbolic descriptions of $\mathcal{O}$ and $\mathcal{O}_F$ are of course identical. The first term in eq.10 corresponds to the second *and* the fourth term of eq.12. Similarly, the second term of eq.10 corresponds to the first *and* the third term of eq.12. Examples below show that this is indeed the case whether effects of gluon condensates are included or not. One can in fact choose to consider only "factorizable" contributions with different vertices and prefactors (first and third term of eq.12 and first term of eq.10).

If the external field is turned off, one sees from the colour structure that the first term in eq.10 corresponds to the second and fourth term in eq.12:

$$N_c^2 = \frac{1}{N_c}N_c + 2Tr(t^a t^a). \tag{13}$$

Similarly, the second term in eq.10 corresponds to the first term (and a vanishing third term) in eq.12.

In order to calculate non-perturbative contributions to B_K, we need fermion propagators in an external gluon field. A pedagogical presentation for this calculation is presented by Novikov et al. in [8]. In the point gauge the gluon field can be expanded in terms of the gluon tensor $G_{\beta\alpha}$:

$$A_\alpha(x) = \frac{1}{2}x^\beta G_{\beta\alpha}(0) + \frac{1}{3}x^\mu x^\beta[D_\mu G_{\beta\alpha}](0) + \frac{1}{8}x^\mu x^\nu x^\beta[D_\mu D_\nu G_{\beta\alpha}](0) + \cdots. \tag{14}$$

When calculating the fermion propagator in an external gluon field we see that we get contributions from "G", "DG" and "DDG" terms. In order to obtain gluon condensate contributions one has interpret products of gluon tensors as vacuum averages. An averaging over gluon indices yields for example the substitution

$$G^a_{\alpha\beta}G^b_{\mu\nu} \to \frac{\delta^{ab}}{96}\langle G^2\rangle(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\nu}). \tag{15}$$

This is how non-perturbative effects are interpreted from a perturbative prescription. Similar expressions as in eq.15, albeit more complicated, can be obtained with three gluons and terms including derivatives. They can be derived by making use of the equations of motion

$$D^\mu G^a_{\mu\nu} = -g_s j^a_\nu, \tag{16}$$

the commutation relation

$$[D_\mu, D_\nu] = -ig_s G_{\mu\nu}, \tag{17}$$

the Bianchi identity and translational invariance. This will enable us to separate the $\langle G^3\rangle$ contribution from $\langle j^2\rangle$. We find the relations

$$\langle 0|(D_\alpha G_{\rho\mu})^a(D^\alpha G^{\rho\mu})^a|0\rangle = 2g_s^2\langle j^2\rangle - 2g_s\langle G^3\rangle \tag{18}$$

$$\langle 0|(D_\alpha G_{\rho\mu})^a(D^\rho G^{\alpha\mu})^a|0\rangle = g_s^2\langle j^2\rangle - g_s\langle G^3\rangle \tag{19}$$

$$\langle 0|(D_\alpha G^{\alpha\mu})^a(D_\rho G^\rho{}_\mu)^a|0\rangle = g_s^2\langle j^2\rangle \tag{20}$$

See also reference [9].

When calculating the off-diagonal $K^0 - \overline{K}^0$ amplitude with external gluons, one can take advantage of the diagrams already included in the expression for f_π. A more refined version of eq.7 including gluon condensates can be written [4]

$$f_\pi^2 = \frac{N_c M^2}{4\pi^2} \left[\hat{f}_\pi + \frac{\pi^2}{6 N_c M^4} \langle \frac{\alpha_s}{\pi} G^2 \rangle + \frac{1}{360 N_c} \frac{\langle g_s^3 G^3 \rangle}{M^6} + \cdots \right] \tag{21}$$

where the dots denotes higher order condensates. During the calculation we need some useful trace expressions of the $SU(3)$ generators:

$$Tr(t^a t^b) = \frac{1}{2} \delta^{ab}, \tag{22}$$

$$Tr(t^a t^b t^c) = \frac{1}{4} (d^{abc} + i f^{abc}) \tag{23}$$

and

$$Tr(t^a t^b t^c t^d) = \frac{1}{4 N_c} \delta^{ab} \delta^{cd} + \frac{1}{8} (d^{abe} + i f^{abe})(d^{cde} + i f^{cde}). \tag{24}$$

When dressing the "blobs" from the $\mathcal{O}_F$ operator with two external gluons ending in vacuum, one can realize that the two diagrams having only one single colour line, i.e. the second and fourth term of eq.12, indeed cancels in the two gluon case (one gluon on each loop) as seen from the colour structure

$$\frac{1}{N_c} Tr(t^a t^b) + 2 Tr(t^a t^c t^b t^c) = 0. \tag{25}$$

In addition, the first term in eq.12 vanishes due to traceless colour matrices. Thus, we only have to consider the diagrams coming from the third term in eq.12 with external gluons attached. This is a non-factorizable contribution due to the vertex structure $\gamma^\mu L t^a$. Non-perturbative gluons to order $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ contributes to B_K as

$$B_K = \frac{3}{4} \left\{ 1 + \frac{1}{N_c} \left[1 - \frac{N_c \langle \frac{\alpha_s}{\pi} G^2 \rangle}{32 \pi^2 f_\pi^4} \right] \right\}. \tag{26}$$

This is in accordance with [4]. The terms with two gluons on each loop are only contributing to f_π and therefore bring no effect into B_K. One should also have in mind that eq.26 is further modified by meson loops [10].

In the case of three gluons, one again realizes that the second and fourth term in eq.12 cancels in the case of two external gluons on one loop and one gluon on the other. The cancellation is again seen by looking at the colour structure. One encounters the sum

$$\frac{1}{N_c} Tr(t^a t^b t^c) + 2 Tr(t^a t^b t^d t^c t^d) = 0 \tag{27}$$

where we have used the commutator and completeness relation of two $SU(3)$ generators and applied the trace expressions for up to four $SU(3)$ generators. The diagrams with all three gluons attached to one loop are already included in the physical pion decay constant. Thus, we are left only with the third term in eq.12 which we dress with gluons. Due to the different "G", "DG" and "DDG" parts in eq.14 there are totally 20 diagrams to calculate.

A useful tool for the calculation is to use a software package for algebraic manipulation. We use the Form program, invented by J. Vermaseren [11], which is well suited for our purpose. By summing up all diagrams, we find zero contribution of the $\langle G^3 \rangle$ terms. This result is a generalization of a result obtained in the paper of W. Hubschmid and S. Mallik [12] where they show that there are no contribution of $\langle G^3 \rangle$ to two-point functions. A more complicated structure is encountered in our

case when calculating the B_K parameter and their result is therefore not automatically applicable in our case.

As a cross-check of our result, we also performed the calculation with just the original operator $\mathcal{O}$. This was a more time consuming operation, but the same vanishing result was obtained.

We have also calculated the contributions proportional to $\langle j^2 \rangle$ and obtained a non-vanishing constant term (constant with respect to the kaon momentum) which is not acceptable. However, $\langle G^4 \rangle$ type contributions should be included as well according to eq.2. To complete this part of the calculation would require a major effort.

4 Conclusion

We have calculated the non-factorizable $< G^3 >$ contribution to the B_K parameter. A vanishing result is obtained as [12] did for the two point function. The next order contributions are coming from operators of dimension 8. To calculate their contribution is a quite extensive task. Also, due to the lack of numerical information on dimension 8 operators, one may wonder if that is worth while at the present stage.

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